EXERCISES [MAI 2.9]

TRANSFORMATIONS OF FUNCTIONS

SOLUTIONS

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A. Paper 1 questions (SHORT)

1.

$y = f(x) + 5$	(1, 5.5)	$y = f(x + 5)$	(-4, 0.5)
$y = f(x) - 5$	(1, -4.5)	$y = f(x - 5)$	(6, 0.5)
$y = 5f(x)$	(1, 2.5)	$y = f(5x)$	(0.2, 0.5)
$y = f(x)/5$	(1, 0.1)	$y = f(x/5)$	(5, 0.5)
$y = -f(x)$	(1, -0.5)	$y = f(-x)$	(-1, 0.5)

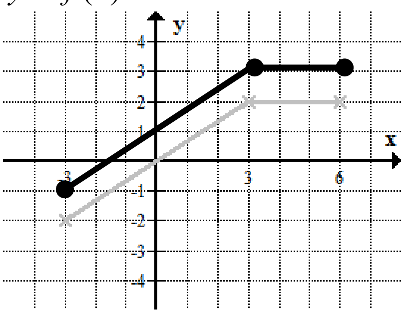
2. (a)

$y = f(x) + 3$	(-1, 6)	$y = f(x + 3)$	(-4, 3)
$y = f(x) - 3$	(-1, 0)	$y = f(x - 3)$	(2, 3)
$y = 3f(x)$	(-1, 9)	$y = f(3x)$	(-1/3, 3)
$y = f(x)/3$	(-1, 1)	$y = f(x/3)$	(-3, 3)
$y = -f(x)$	(-1, -3)	$y = f(-x)$	(1, 3)

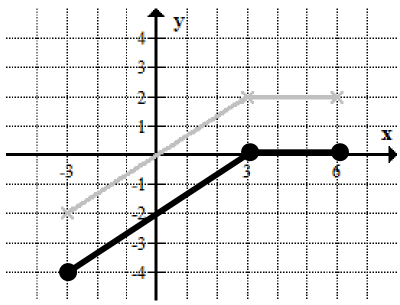
(b)

$f(x)$ (-1, 3)
$2f(x)$ (-1, 6)
$2f(x\text{-}3)$ (2, 6)
$2f(x\text{-}3)+4$ (2, 10) Hence the corresponding point is (2,10)

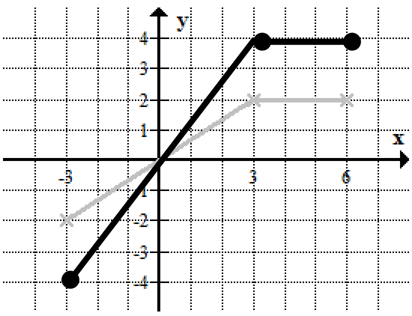
3. (a) $y = f(x) + 1$



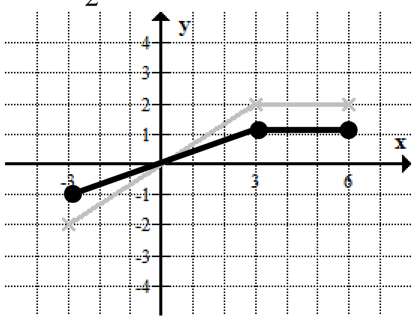
(b) $y = f(x) - 2$



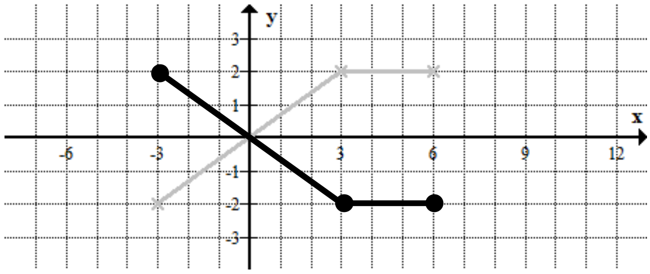
(c) $y = 2f(x)$



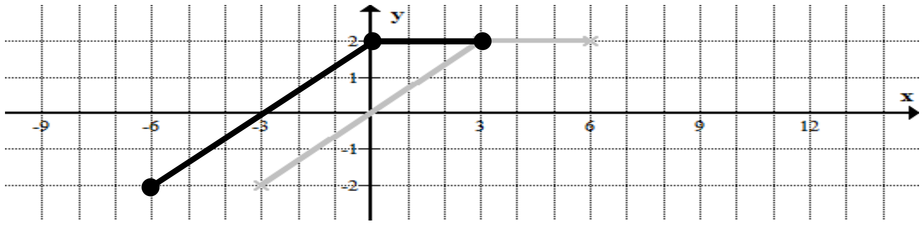
(d) $y = \frac{1}{2} f(x)$



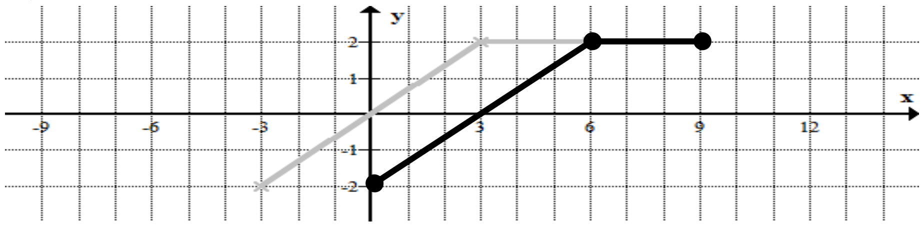
(e) $y = -f(x)$



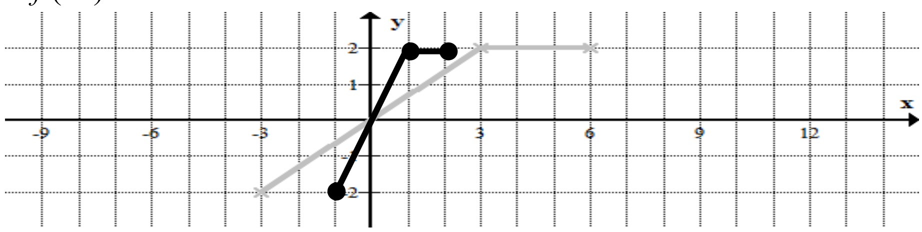
(f) $y = f(x+3)$



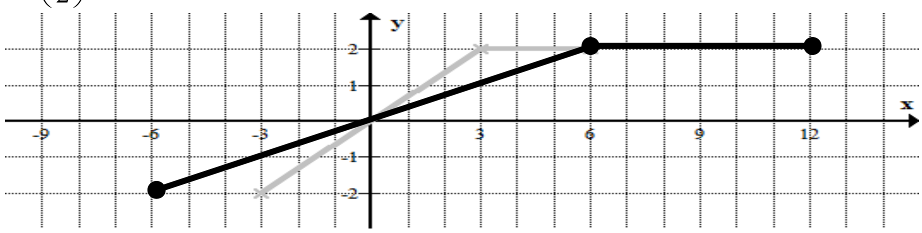
(g) $y = f(x-3)$



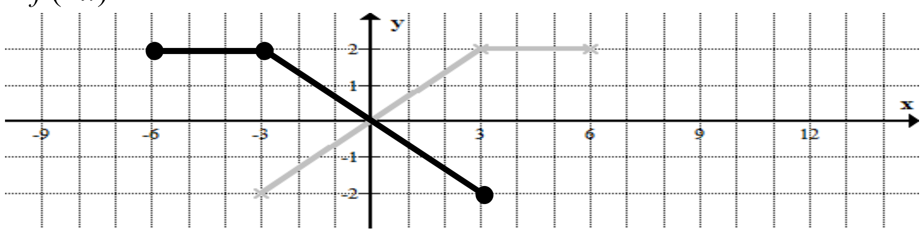
(h) $y = f(3x)$



(i) $y = f\left(\frac{x}{2}\right)$



(j) $y = f(-x)$



4. (a) $-f(x-2)+5$

$f(x)$	original
$-f(x)$	reflection in *x*-axis
$-f(x-2)$	horizontal translation 2 units to the right
$-f(x-2)+5$	vertical translation 5 units up

(b) $-3f(x+2)-1$

$f(x)$	original
$-f(x)$	reflection in *x*-axis
$-3f(x)$	vertical stretch with s.f. 3
$-3f(x+2)$	horizontal translation 2 units to the left
$-3f(x+2)-1$	vertical translation 1 unit down

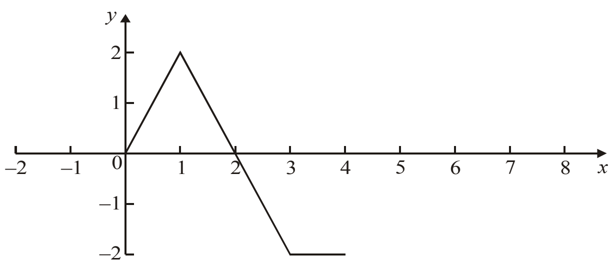
(c) $f(2x-10)$

$f(x)$	original
$f(x-10)$	horizontal translation 10 units to the right
$f(2x-10)$	horizontal stretch with s.f. 1/2 (i.e. shrink)

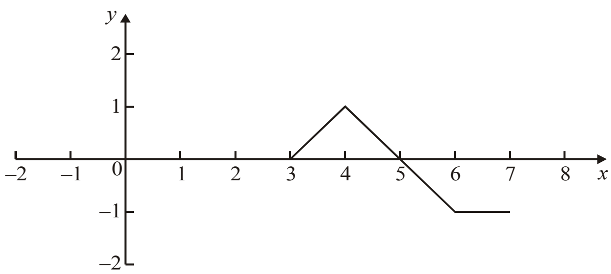
(d) $f(2(x-5))$

$f(x)$	original
$f(2x)$	horizontal stretch with s.f. 1/2 (i.e. shrink)
$f(2(x-5))$	horizontal translation 5 units to the right

5. (a) (i)



(ii)



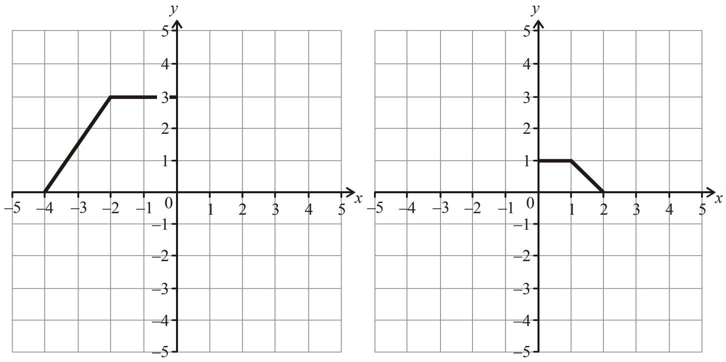
(b) A′ (3, 2) (Accept $x=3$, $y=2$)

6. (a) (I) D (ii) C (iii) A

(b) B: $f(x)+2$ E: $f(x\text{-}2)$

7. (a) (i) 1 (ii) 0.5

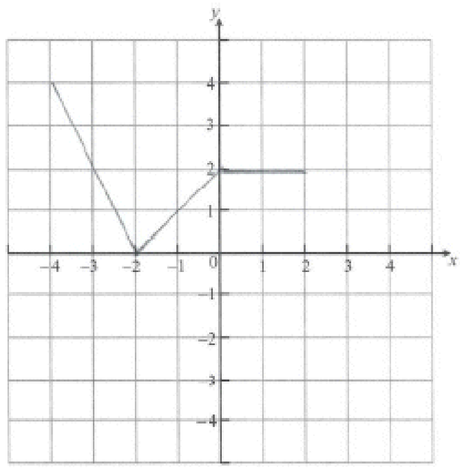
(b)



(c)

	$y = f(x)$	$y = 3f(-x)$	$y = f(2x)$.
Domain	$0 \le x \le 4$	$-4 \le x \le 0$	$0 \le x \le 2$
Range	$0 \le y \le 1$	$0 \le y \le 3$	$0 \le y \le 1$

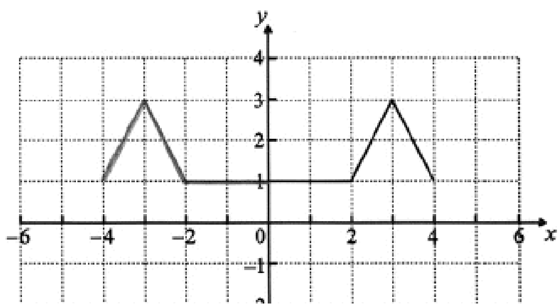
8. (a)



(b) $x = 3 + 1, y = \frac{1}{2} \times 2$

P is (4, 1) (accept $x = 4$, $y = 1$)

9. (a)



(b)

Description of transformation	Diagram letter
Horizontal stretch with scale factor 1.5	**C**
Maps f to $f(x) + 1$	**D**

(c) translation (move/shift/slide *etc.*) 6 units to the left and 2 units down

10. (a) By GDC the coordinates are (-1,1.66) [or $\left(-1, \frac{5}{3}\right)$]

[Notice: it can also be found by using derivatives later on]

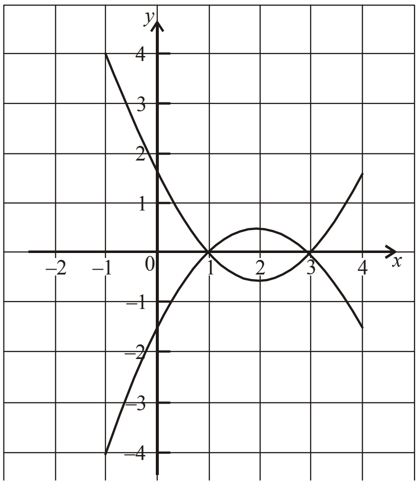
(b) (i) (–3, –9)

(ii) (1, –4)

(iii) reflection gives (3, 9)

stretch gives $\left(\frac{3}{2}, 9\right)$

11. (a)



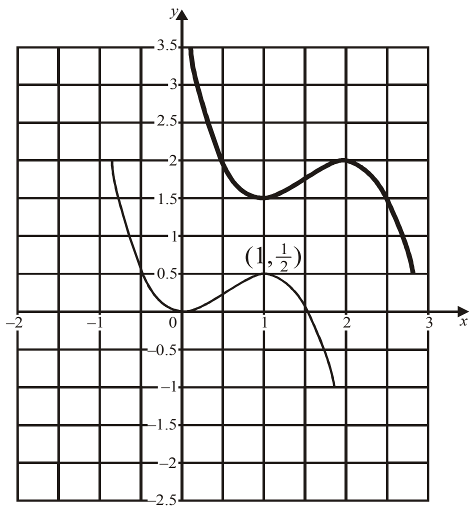
(b) (i) $g(-3) = f(0)$ $f(0) = -1.5$

(ii) translation (accept shift, slide, *etc.*) of $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

(c)

	$y = f(x)$	$y = -f(x)$	$y = f(x+3)$.
Domain	$-1 \le x \le 4$	$-1 \le x \le 4$	$-4 \le x \le 1$
Range	$-4 \le y \le 0.5$	$0.5 \le y \le 4$	$-4 \le y \le 0.5$

12. (a)



(b) Minimum: $\left(1, \frac{3}{2}\right)$ Maximum: (2, 2)

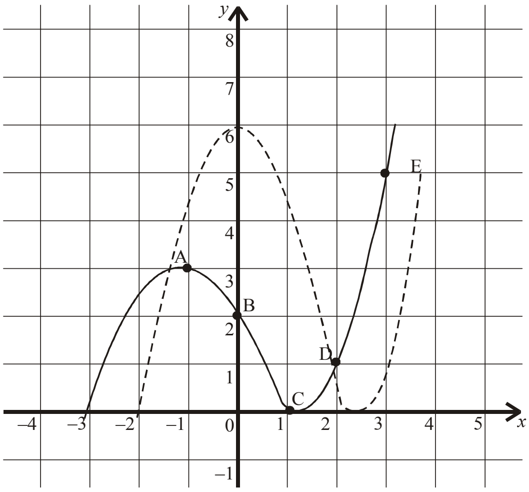
13. (a) $g(x) = 2f(x-1)$

x	0	1	2	3
$x-1$	-1	0	1	2
$f(x-1)$	3	2	0	1

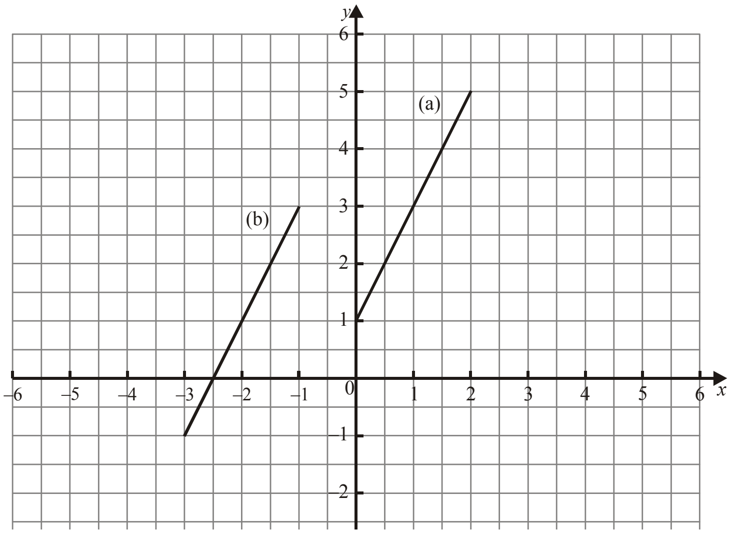
$g(0) = 2f(-1) = 6$ $g(1) = 2f(0) = 4$

$g(2) = 2f(1) = 0$ $g(3) = 2f(2) = 2$

(b) Graph passing through (0, 6), (1, 4), (2, 0), (3, 2)



14. (a) and (b) (Mind the endpoints)



(c)

	$y = f(x)$	$y = g(x)$
Domain	$0 \le x \le 2$	$-3 \le x \le -1$
Range	$1 \le y \le 5$	$-1 \le y \le 3$

15. (a) $y = (x-1)^2$

$y = 4(x-1)^2$

$y = 4(x-1)^2 + 3$

(b)

$y = x^2$	(0,0)
$y = (x\text{-}1)^2$	(1,0)
$y = 4(x\text{-}1)^2$	(1,0)
$y = 4(x\text{-}1)^2+3$	(1,1)

16. (a) in any order
translated 1 unit to the right
stretched vertically by factor 2

(b) **METHOD 1**

Finding coordinates of image on g

$(-1, 1) \rightarrow (-1 + 1, 2 \times 1), (0, 2)$ then P is (3, 0)

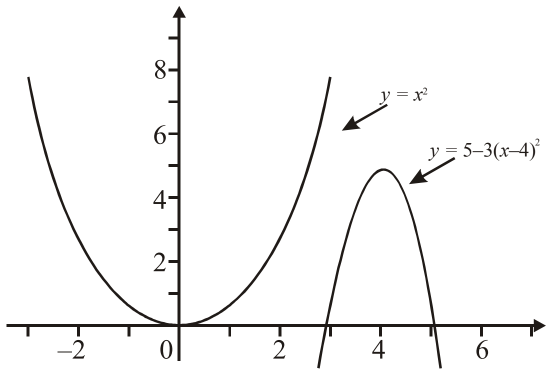
METHOD 2

$h(x) = 2(x - 4)^2 - 2$ P is (3, 0)

17. (a) (1, – 2)
(b) $g(x) = 3(x - 1)^2 - 2$ (accept $p = 1$, $q = -2$)
(c) (1, 2)

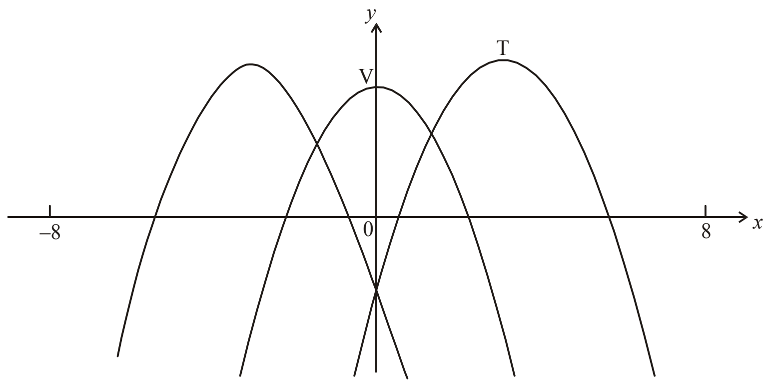
18. (a) $y = 2(x - 3)^2 + 5$ (accept $c = 3$, $d = 5$)
(b) (i) $k = 2$ (ii) $p = 3$ (iii) $q = 5$

19.



$q = 5$ $k = 3, p = 4$

20. (a) (i) $h = 3$ (ii) $k = 1$
(b) $g(x) = f(x - 3) + 1$, $5 - (x - 3)^2 + 1$, $6 - (x - 3)^2$, $-x^2 + 6x - 3$
(c)



reflect through y-axis, vertex at approximately (– 3, 6).

21. (a) $3(x - 2)^2 - 1$ (Accept $h = 2$, $k = 1$)

(b) **METHOD 1**
Vertex shifted to $(2 + 3, -1 + 5) = (5, 4)$
so the new function is $3(x - 5)^2 + 4$ (Accept $p = 5$, $q = 4$)

METHOD 2
$g(x) = 3((x - 3) - h)^2 + k + 5 = 3((x - 3) - 2)^2 - 1 + 5$
$= 3(x - 5)^2 + 4$ (Accept $p = 5$, $q = 4$)

B. Paper 2 questions (*LONG*)

22. (a) attempt to form composition (in any order)

$(f \circ g)(x) = (x-1)^2 + 4 \quad (x^2 - 2x + 5)$

(b) **METHOD 1**

vertex of $f \circ g$ at (1, 4)

adding $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to the coordinates

vertex of h at (4, 3)

METHOD 2

$h(x) = (x-4)^2 + 3$

vertex of h at (4, 3)

(c) $h(x) = x^2 - 8x + 19$

(d) equating functions to find intersection point: $x^2 - 8x + 19 = 2x - 6$

$x^2 - 10x + 25 = 0$

$x = 5$

OR find the point of intersection P(5,4) by using graphs.

(e) $x^2 - 8x + 19 = 2x - 5$

Use graphs to obtain the intersection points (4,3) and (6,7)

23. (a) $f(x) = 3(x^2 + 2x + 1) - 12 = 3x^2 + 6x + 3 - 12 = 3x^2 + 6x - 9$

(b) (i) vertex is $(-1, -12)$

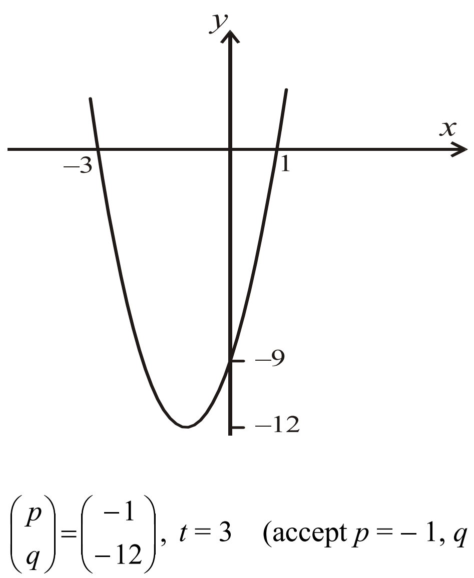
(ii) $x = -1$ (**must** be an equation)

(iii) $(0, -9)$

(iv) solving $f(x) = 0$

$(-3, 0), (1, 0)$

(c)



(d) $\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 \\ -12 \end{pmatrix}$, $t = 3$ (accept $p = -1$, $q = -12$, $t = 3$)